Date 21/11/23

saathi

Chapter - Introduction to three Dimensional Geometry

Octant

	I	II	III	IV	V	VI	VII	VIII
x	+	-	-	+	+	-	-	+
y	+	+	-	-	+	+	-	-
z	+	+	+	+	-	-	+	-

Exercie = 12.1

1. A point - - - - - - - - - - - - z-coordinate?

point lie on x axis
then y coordinate = 0
z coordinate = 0 (Ans)

2. A poin lie on XZ-plane
then y = 0 (Ans)

3.

Point	Octant
(1,2,3)	I
(4,-2,3)	IV
(4,-2,-5)	VIII
(4,2,-5)	V
(-4,2,-5)	VI
(-4,2,5)	II
(-3,-1,6)	III
(-2,-4,-7)	VII

4.
(i) ~~X=Y~~ XY-plane
(ii) (x, y, 0)
(iii) 8 octants

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$A(x_1, y_1, z_1)$ $B(x_2, y_2, z_2)$

$$AB = d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

Exercise - 12.2

1.

(i) $A(2, 3, 5)$ $B(4, 3, 1)$
$x_1\ y_1\ z_1$ $x_2\ y_2\ z_2$

$$AB = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

$$= \sqrt{(2-4)^2+(3-3)^2+(5-1)^2}$$

$$= \sqrt{(-2)^2+(0)^2+(4)^2}$$

$$= \sqrt{4+16} \Rightarrow \sqrt{20} \Rightarrow 2\sqrt{5} \text{ unit (Ans)}$$

(iii) $A(-1, 3, -4)$ $B(1, -3, 4)$
$x_1\ y_1\ z_1$ $x_2\ y_2\ z_2$

$$AB = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

$$\sqrt{(-1-1)^2+(3+3)^2+(-4-4)^2}$$

$$\sqrt{(-2)^2+(6)^2+(-8)^2}$$

$$\sqrt{4+36+64} \Rightarrow \sqrt{104} \Rightarrow 2\sqrt{26}$$

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3. A(0, 7, -10) B(-1, 6, -6) C(4, 9, -6)

$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

$\sqrt{(0+1)^2 + (7-6)^2 + (10-6)^2}$

$\sqrt{(1)^2 + (1)^2 + (4)^2}$

$\sqrt{1 + 1 + 16} \Rightarrow \sqrt{18}$

BC $\sqrt{(-1+4)^2 + (6-9)^2 + (6-6)^2}$

$\sqrt{(3)^2 + (-3)^2 + (0)^2}$

$\sqrt{9 + 9 + 0} \Rightarrow \sqrt{18}$

CA $\sqrt{(-4-0)^2 + (9-7)^2 + (6-10)^2}$

$\sqrt{(-4)^2 + (2)^2 + (-4)^2}$

$\sqrt{16 + 4 + 16} \Rightarrow \sqrt{36} \Rightarrow 6$

$AB^2 + BC^2 = CA^2$

$(\sqrt{18})^2 + (\sqrt{18})^2 = (6)^2$

$36 = 36$

Hence these vertices are right angle Δ

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2. A $(-2,3,5)$ B $(1,2,3)$ C $(7,0,-1)$

$x_1\ y_1\ z_1$ $x_2\ y_2\ z_2$

AB $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$

$\sqrt{(-2-1)^2+(3-2)^2+(5-3)^2}$

$\sqrt{(-3)^2+(1)^2+(2)^2}$

$\sqrt{9+1+4} \Rightarrow \sqrt{14}$

BC $\sqrt{(1-7)^2+(2-0)^2+(3+1)^2}$

$\sqrt{(6)^2+(0)^2+(4)^2}$

$\sqrt{36+0+16} \Rightarrow \sqrt{52} \Rightarrow 2\sqrt{14}$

CA $\sqrt{(7+2)^2+(0-3)^2+(-1-5)^2}$

$\sqrt{(9)^2+(3)^2+(-6)^2}$

$\sqrt{81+9+36} \Rightarrow \sqrt{126} \Rightarrow 3\sqrt{14}$

$AB+BC = \sqrt{14}+2\sqrt{14} = 3\sqrt{14} = AC$

$AB+BC = AC$

Hence points are Collinear (Ans)

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Q4- Find - - - - - - - - - - - - - - - (1,2,3) and (3,2,-1)

Let points $P(x, y, z)$

A (1,2,3)   B (3,2,-1)

$$d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$$

A.T.Q.

$$PA = PB$$

$$\sqrt{(x-1)^2 + (y-2)^2 + (z-3)^2} = \sqrt{(x-3)^2 + (y-2)^2 + (z+1)^2}$$

S.B.S.

$$x^2 + 1 - 2x + y^2 + 4 - 4y + z^2 + 9 - 6z = x^2 + 9 - 6x + y^2 + 4 - 4y + z^2 + 1 + 2z$$

$$\Rightarrow -2x - 6z + 6x - 2z = 0$$

~~$-8z$~~ $4x - 8z = 0$

$$4(x - 2z) = 0$$

$$x - 2z = 0 \quad \text{(Ans)}$$

5- Find - - - - - - - - - - - - - - - - is equal to 10.

Let point $P(x, y, z)$

$A(4, 0, 0)$ $B(-4, 0, 0)$

$$d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$$

A.T.Q.

$$PA + PB = 10$$

$$\sqrt{(x-4)^2 + (y-0)^2 + (z-0)^2} + \sqrt{(x+4)^2 + (y-0)^2 + (z-0)^2} = 10$$

$$\sqrt{(x-4)^2 + y^2 + z^2} = 10 - \sqrt{(x+4)^2 + y^2 + z^2}$$ ~~= 10~~

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S.B.S.

$$\left[\sqrt{(x-4)^2+y^2+z^2}\right]^2 = \left[10-\sqrt{(x+4)^2+y^2+z^2}\right]^2$$

$$(x-4)^2+y^2+z^2 = 100+(x+4)^2+y^2+z^2-20\sqrt{(x+4)^2+y^2+z^2}$$

$$x^2+16-8x = 100+x^2+16+8x-20\sqrt{(x+4)^2+y^2+z^2}$$

$$-8x-8x-100 = -20\sqrt{(x+4)^2+y^2+z^2}$$

$$-16x-100 = -20\sqrt{(x+4)^2+y^2+z^2}$$

$$\pm 4(4x+25) = \pm 20\sqrt{(x+4)^2+y^2+z^2}$$

$$4x+25 = 5\sqrt{(x+4)^2+y^2+z^2}$$

S.B.S

$$(4x+25)^2 = 5$$

$$16x^2+625+200x = 16x^2+625 = 25(x+4)^2+y^2+z^2$$

$$16x^2+625+200x = 25(x^2+16+8x+y^2+z^2)$$

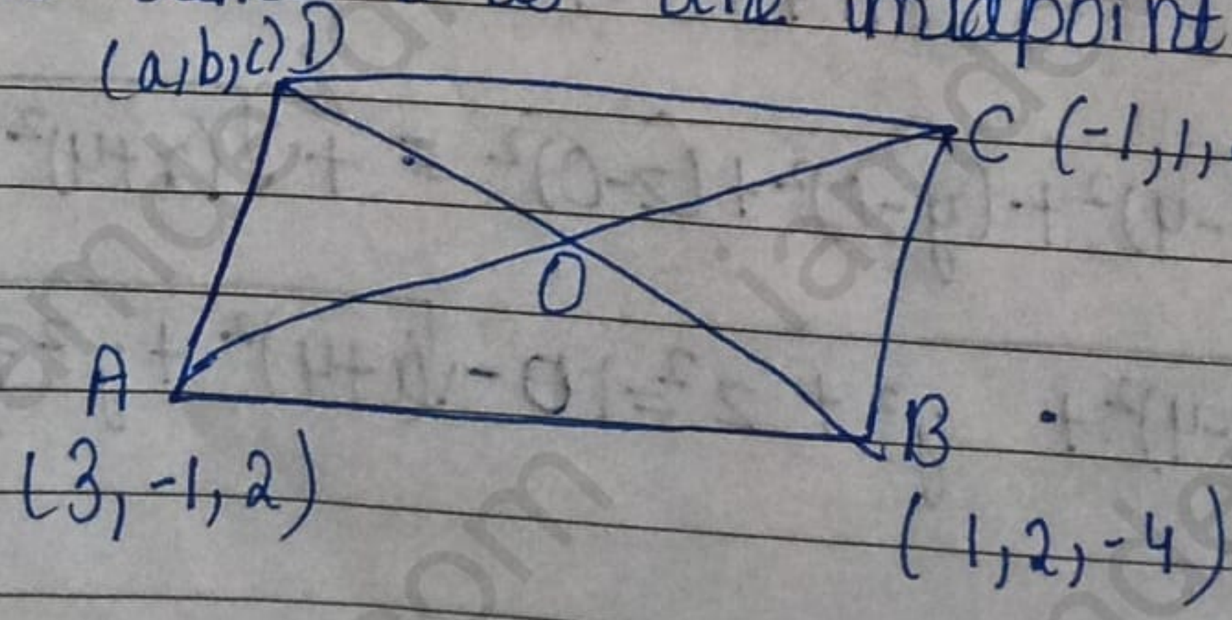
$$0 = 25x^2+25y^2+25z^2+200x+400-16x^2-625-200x$$

$$\Rightarrow \boxed{9x^2+25y^2+25z^2-225=0} \text{ (Ans)}$$

Miscellaneous Exercise

3. 3 vertices - - - - - - - - - - - - - - - - vertex.

We know that the diagonal of a parallelogram bisect each other hence O is the midpoint of AC



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Mid point $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

$O\left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right)$

$O\left(\frac{2}{2}, \frac{0}{2}, \frac{4}{2}\right)$ -(i)

O is the mid point of BD also O

$O\left(\frac{a+1}{2}, \frac{b+2}{2}, \frac{(-4)}{2}\right)$ -(ii)

from (i) and (ii)

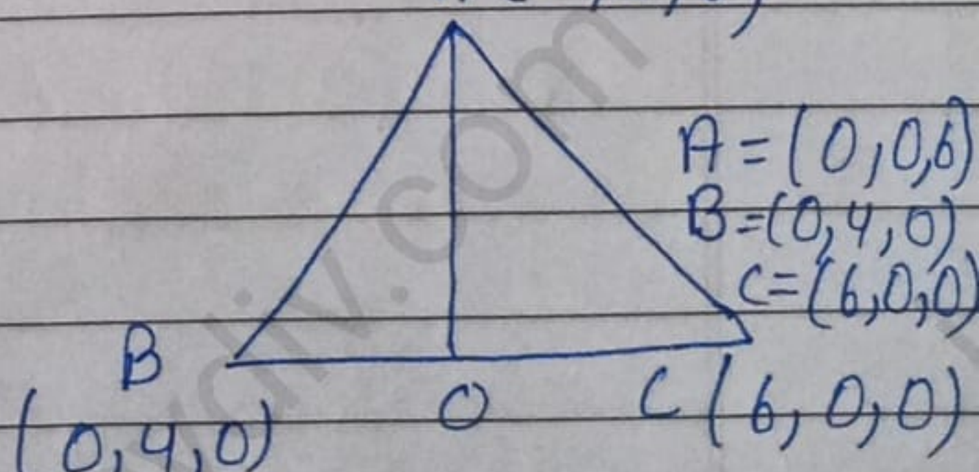
$\frac{a+1}{2} = \frac{2}{2} \Rightarrow a+1=2 \Rightarrow \boxed{a=1}$

$\frac{b+2}{2} = \frac{0}{2} \Rightarrow b+2=0 \Rightarrow \boxed{b=-2}$

$\frac{C-4}{2} = \frac{4}{2} \Rightarrow C-4=4 \Rightarrow \boxed{C=8}$

Vertex of IV th point (1, -2, 8)

Q2- Find the lengths - - - - - - - - - - - (6,0,0)



AD is a median

D is a midpoint of BC

$D\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

$D\left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2}\right)$

D(3,2,0)

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$A(0,0,6)$ $D(3,2,0)$

$$AD = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$$

$$= \sqrt{(0-3)^2 + (0-2)^2 + (6-0)^2}$$

$$= \sqrt{(-3)^2 + (-2)^2 + (6)^2}$$

$$= \sqrt{9+4+36} = \sqrt{49}$$

$AD = 7$ units (Ans)

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